An Aesthetic Curve in the Field of Industrial Design

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Abstract

We present a quantitative analysis method of the characteristics of a curve, for finding what an aesthetic curve is. A number of designer drawn curves were analyzed by this method. As a result, we found that the designer controlled the curvature change with a self-affine property, when he produced a curve in design work. In other words, the designer sees a curve with a self-affine property as an aesthetic curve. On the basis of this fact, we developed five types of curves which have the self-affine property. Furthermore, we made 'drawing-curves' from these curves as 'visual language'. These 'drawing-curves' can be used as a 'common language' between the designer, the modeler, and the operator of CAD systems for communicating the 'design intent'.

1. Introduction

A curve is an important design element to define the overall shape of a product. And, drawing an aesthetic curve is a requirement for a good designer.

In the field of engineering, the representation of a curve on a computer has been studied, e.g., the curve to use a parametric function was presented by Ferguson, Coons, and the like, and a spline function was presented by Bézier, Gordon, Riesenfeld, and the like [1, 2, 3, 5]. However, in all these studies, a set of given points that forms the locus of the curve was assumed "absolute". In other words, the studies required that the designer define the precise locus of the curve by a set of points.

In general, a designer wants to make a set of points as smooth as possible, having some "rhythm". The set of points is made rather *roughly* by hand, because a designer can't control the points so precisely. Most earlier studies on engineering discuss making a curve only "smooth" in some mathematical sense through or near these rough points. Therefore, the curve has no "rhythm" in most cases.

A few studies [4, 8, 10, 11, 12] discussed what characteristics made an aesthetic curve. For example, Pal and Nutbourne presented an algorithm for the generation of a smooth curve through two data points with specified curvature and tangent directions at those points. They argued that fair curve needed curvature profile was designed to be piecewise linear, constructed from linear elements. On the basis of this fact, they said that a fair curve form consisted of arcs of Cornu spirals (clothoids). Further, Farouki presented an algorithm for the generation of fair curve by the use of Pythagorean-hodograph (PH) quintic transition curves.

In this paper, we develop a method for analyzing what characteristics made an aesthetic curve [6]. This method use a relation between a length frequency of the curve and a radius of the curvature in a log-log coordinate system. By using our method, we analyze many sample curves on the products (drafts) made and drawn by designer's hands. As a result of this, we clarify that there are a lot of curves that we can not represent by only arcs of Cornu spirals and PH spirals, and the designer control the curvature change with a self-affine property. Second, on the basis of the result of this analysis, we develope an algorithm for generating a curve having self-affine properties. Then, by using this algorithm, we can get not only ' decelerating' curves like spirals but also 'accelerating' curves by same parameters. There are not an algorithm for getting 'accelerating' curves and 'decelerating' curves uniformly, yet. And we get a new criterion for fairness of curve by self-affine properties. In this way, our viewpoint is different from earlier ones.

Moreover, on the basis of this fact, we systematize

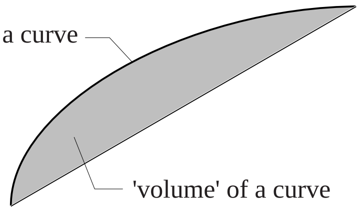

Figure 1. Volume of a curve.

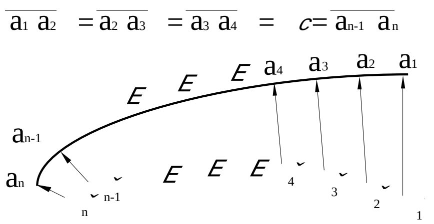

Figure 2. A method of extracting constitutional points.

aesthetic curves, and attempt to make 'drawing-curves' from these curves as 'visual language'. We apply these 'drawing-curves' to industrial design.

2. Quantitative analysis method

In this section we propose a quantitative analysis method for the characteristics of a curve. We suppose that the curve treated in this study, satisfies the following 4 conditions: it is 1)plane, 2)open, 3)non-intersecting, and 4)monotonic.

We analyzed the design words expert car designers used to find what characteristics of a curve they pay attention to. We found some words that expressed the characteristics of a curve. Furthermore, from a viewpoint of mathematics, the words were concerned with curvature change and volume of a curve. Here, 'volume' is defined as the area bound by the curve and the straight line binding the starting point and the ending point of a curve (see Figure 1). Thus, we define that the characteristics of a curve means its curvature change and volume in this study. In fact, when curvature change and volume are defined, the curve is fixed reversely.

We developed a method for analyzing the curvature change and the volume of curves mathematically, simultaneously and intuitively. In this method, we interpolate a sample curve using a Bézier curve on a computer and express the relation between the radius

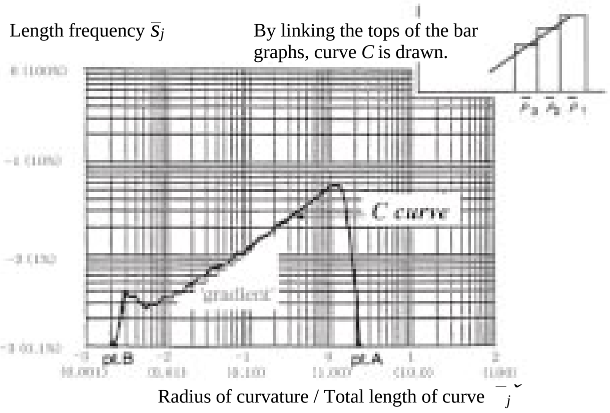

Figure 3. Logarithmic distribution diagram of curvature.

of curvature at every constitutional point on the interpolated curve and the "length" of the curve showing the total length of segmental curves corresponding to this radius of curvature in a log-log coordinate system. This is called a "logarithmic distribution diagram of curvature". Now, let us explain it in detail as follows.

First, let us denote the total length of the curve by S_{all}, the length of a segmental curve by s_j, the radius of curvature at a constitutional point a_i by ρ_i, and the interval of the radius of the curvature by $\bar{\rho}_j$ (the units are mm in all cases.). The radius of the curvature ρ_i at the constitutional point a_i on the curve is obtained by extracting constitutional points $(a_1, a_2, \cdots, a_n)$ at equal intervals as shown in Figure 2 (e.g., $S_{all} = 100$mm, and the constitutional points were extracted at 0.1mm intervals in actual dimension, therefore, n=1000), and by calculating the respective radii of the curvature $(\rho_1, \rho_2, \cdots, \rho_n)$ at the respective constitutional points.

Second, let us denote the interval of the radii of the curvature $\bar{\rho}_j$ by the interval corresponding to the quotient obtained by dividing the common logarithm $[-3, 2]$ of the value ρ_i/S_{all} $[0.001, 100]$ by 100 equally (determined by surveying the range of the curves adopted into actual cars). In other words, $\bar{\rho}_m = [-3 + 0.05(m-1), -3 + 0.05m]$ (m is an integer between 1 and 100, i.e., $1 \leq m \leq 100$).

Third, the numbers of occurrence of the common logarithm values of $\rho_1/S_{all}, \rho_2/S_{all}, \cdots, \rho_n/S_{all}$ in each interval of $\bar{\rho}_j$ is summed up. From this number $(=N_j)$, the length of the segmental curve s_j (=distance between neighboring constitutional points $\times N_j$) in which the $\bar{\rho}_j$ appears, is calculated. This means that $S_{all} = s_1 + s_2 + \cdots + s_{100}$. In addition, "length frequency" $\bar{s}_j [= \log_{10}(s_j/S_{all})]$ was defined for representing the

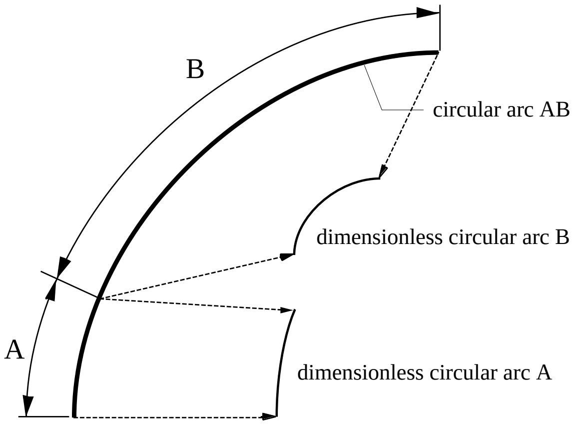

Figure 4. Two dimensionless circular arcs A and B.

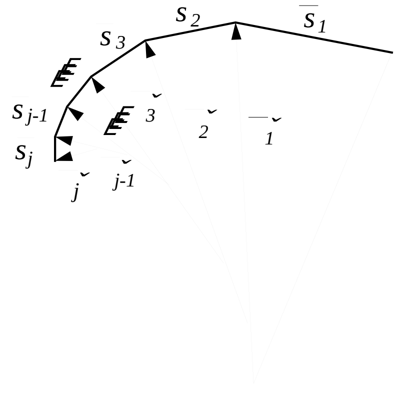

Figure 5. The relation of $\bar{\rho}_j$ and $\bar{s}_j$.

ratio of the length of a segmental curve to the total length of the curve S_{all}.

The "logarithmic distribution diagram of curvature" defined above can be obtained by taking $\bar{\rho}_j$ for the horizontal axis and $\bar{s}_j$ for the vertical axis, as shown in Figure 3. In this figure, numeric values of ρ_i/S_{all} (magnification) and s_j/S_{all} (%) are shown also in parenthesis for convenience of understanding. To draw such a "logarithmic distribution diagram of curvature" means mathematically obtaining a locus of $d\bar{s}/d\bar{\rho}$ in terms of the interval of the radii of curvature, $\bar{\rho}$, and the length frequency, $\bar{s}$.

In addition, the horizontal axis shows the interval of the radius of the curvature $\bar{\rho}$, which is the radius of the curvature ρ made dimensionless by dividing by the total length S_{all} of the curve. The reason for this can be explained as follows. If two circular arcs A and B having the same curvature but different lengths as shown in Figure 4 are shown on the "logarithmic distribution diagram of curvature" without making ρ dimensionless by S_{all}, the locus of $d\bar{s}/d\bar{\rho}$ for both A and B will be shown at the same position. However, the designer distinguishes the difference between the volumes of dimensionless circular arcs A and B. Therefore, by using ρ made dimensionless by S_{all} for the horizontal axis and drawing the locus of $d\bar{s}/d\bar{\rho}$, the positions on the "logarithmic distribution diagram of curvature" become different according to the total lengths of the curves even if they have the same curvature, showing the difference in volume of both curves visually.

In this "logarithmic distribution diagram of curvature", the way the curvature changes, is shown by the locus of *C curve* shown in Figure 3, and the volume of the curve is shown by the point A and the distance between the two points A and B as shown in Figure 3. Here, if that distance is shorter, the volume of the curve becomes larger. According to the volume, if the curve is interpolated by Bézier curves, the exact value of the volume can be obtained directly by integrating the interpolated curve. Therefore, this value is used for getting an accurate value of the volume. In the quantitative analysis described in this paper, however, the definition of the volume mentioned above is used as an index of relative change of the volume showing the volume change among the sections when the plural cross sections of a curved surface are to be analyzed simultaneously.

Furthermore, the "gradient" of *C curve* in Figure 3 is defined by

$$\begin{aligned}\text{"gradient"} = d\bar{s}/d\bar{\rho} = dY/dX &= \lim_{\Delta X \to 0} \Delta Y/\Delta X \\ &= \lim(Y_{j-1} - Y_j)/(X_{j-1} - X_j) \\ &= \lim(\bar{s}_{j-1} - \bar{s}_j)/(\bar{\rho}_{j-1} - \bar{\rho}_j).\end{aligned}$$

In this paper, the "gradient" means the gradient after transforming the coordinate system to a X-Y rectangular coordinate system with the horizontal axis representing $X = \bar{\rho}_j$ and the vertical axis $Y = \bar{s}_j$. When the "gradient" is a, the relation of the interval of the radii of the curvature $\bar{\rho}_j$ and the length frequency $\bar{s}_j$ is defined by

$$\begin{aligned}\lim(\bar{s}_1 - \bar{s}_2)/(\bar{\rho}_1 - \bar{\rho}_2) = \cdots = \lim(\bar{s}_{j-1} - \bar{s}_j)/(\bar{\rho}_{j-1} - \bar{\rho}_j) \\ = a \quad \text{(as shown in Figure 5).}\end{aligned}$$

Here, if the "gradient" is constant, then the curve has a self-affine property(see Figure 6).

3 "Gradients" of C curves of important functions

In this section, we calculate the "gradients" of *C curves* of important functions in the field of industrial

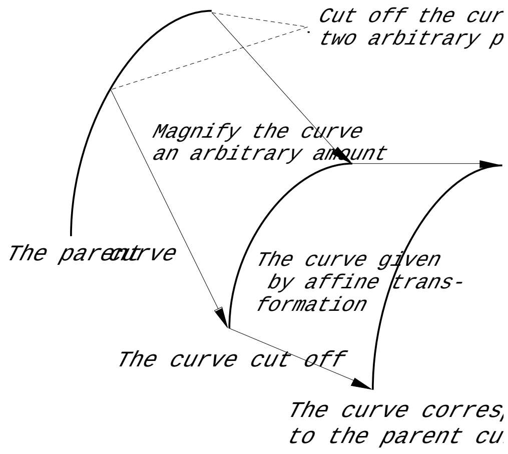

Figure 6. Self-affine property.

design, mathematically. Firstly, let us calculate the "gradient" of the *C curve* of parabola mathematically.
[Parabola]

A parabola is defined by

$$y = ax^2. \tag{1}$$

When a curve $f(x)$ is second times continuously differentiable, the radius of the curvature ρ is given by

$$\rho = \frac{(1 + f'(x)^2)^{3/2}}{|f''(x)|}. \tag{2}$$

Substituting Eq.(1) into Eq.(2), we obtain

$$\rho = \frac{(1 + 4a^2x^2)^{3/2}}{|2a|}. \tag{3}$$

Squaring both sides of Eq.(3), we obtain

$$(2a\rho)^2 = (1 + 4a^2x^2)^3,$$

that is,

$$(2a\rho)^{2/3} = (1 + 4a^2x^2). \tag{4}$$

When x is sufficiently large, $4a^2x^2 \gg 1$, and $1 + 4a^2x^2 \cong 4a^2x^2$. In this case, Eq.(4) is expressed as

$$(2a\rho)^{2/3} = 4a^2x^2. \tag{5}$$

When both sides are expressed logarithmically,

$$2/3(\log 2 + \log a + \log \rho) = \log 4 + \log a + \log ax^2. \tag{6}$$

The length of the segmental curve s in an 'interval of radius of curvature' is defined as

$$s = \int (1 + f'(x)^2)^{1/2} dx, \tag{7}$$

that is,

$$ds/dx = (1 + f'(x)^2)^{1/2}. \tag{8}$$

Substituting Eq.(1) into Eq.(8), we obtain

$$ds/dx = (1 + 4a^2x^2)^{1/2} \cong 2ax, \tag{9}$$

that is,

$$\int ds = \int 2ax dx + c, \tag{10}$$

where c is an arbitrary constant. When x is sufficiently large, the arbitrary constant is negligible. In this case, Eq.(10) is expressed as

$$s = ax^2. \tag{11}$$

Substituting Eq.(11) in Eq.(6), we obtain

$$2/3(\log 2 + \log a + \log \rho) = log4 + loga + logs. \tag{12}$$

By setting $X = \log \rho$, $Y = \log s$, we obtain

$$c1 + c2 + 2/3X = c3 + c4 + Y, \tag{13}$$

that is,

$$Y = \underline{2/3} \times X + C \qquad (C = c1 + c2 - c3 - c4). \tag{14}$$

Here, Figure 3 is a result of the analysis for a parabola. The "gradient" of the *C curve* in Figure 3 is 2/3, which is consistent with the result calculated above.

In a similar way, the "gradients" of *C curves* of sine curve, equiangular spiral, and logarithmic curve, were calculated as follows.
[Sine curve]

A sine curve is defined by

$$y = a \sin x. \tag{15}$$

Since, this curve is second times continuously differentiable, it is possible to calculate the radius of the curvature ρ by Eq.(2). Substituting Eq.(15) into Eq.(2), we obtain

$$\rho = \frac{(1 + a^2 \cos^2 x)^{3/2}}{|-a \sin x|}, \tag{16}$$

that is,

$$\rho a \sin x = (1 + a^2 \cos^2 x)^{3/2}. \tag{17}$$

The length of segmental curve s in an 'interval of radius of curvature' is defined as

$$\int ds = \int (1 + a^2 cos^2 x)^{1/2} dx \tag{18}$$

In the neighborhood of $x = 0$

As a sine curve where $a \gg 0$ is adopted in design generally, we can set $1 + a^2 \cos^2 x \cong a^2 cos^2 x$, and Eq.(18) is expressed as

$$s = \int a \cos x dx = a \sin x. \qquad (19)$$

Substituting Eq.(17) in Eq.(19), we obtain

$$s\rho = (a^2 cos^2 x)^{3/2}. \qquad (20)$$

When both sides are expressed logarithmically,

$$\log(s\rho) = \log(a^2 - a^2 \sin^2 x)^{3/2}. \qquad (21)$$

As $sin^2 x \ll 1$ near $x = 0$, $a^2 - a^2 sin^2 x \cong a^2$,

$$\log s + \log \rho = \frac{3}{2} \log a^2. \qquad (22)$$

By setting $X = \log \rho$, $Y = \log s$, we obtain

$$Y = \underline{-1} \times X + C. \qquad (23)$$

In the neighborhood of $x = \pi/4$

In this neighborhood, $x \cong \pi/4$, and $\cos x \cong 0.70$. As $a \gg 0$, $1 + a^2 \cos^2 x \cong a^2 \cos^2 x$. Eq.(19) can be used as it is. In the same way, Eq.(16) is expressed as

$$\rho = \frac{(a^2 \cos^2 x)^{3/2}}{a \sin x}, \qquad (24)$$

therefore,

$$\log \rho = 3 \log(a \cos x) - \log(a \sin x). \qquad (25)$$

In the neighborhood of $x = \pi/4$, $\sin x \cong \cos x$. Then, we obtain

$$\log \rho = 2 \log(a \sin x), \qquad (26)$$

that is,

$$Y = \underline{1/2} \times X. \qquad (27)$$

In the neighborhood of $x = \pi/2$

As $\cos x \ll 1$, $a \cos x \ll 1$, and $1 + a^2 \cos^2 x \cong 1$. Eq.(18) is expressed as

$$\int ds = \int (1 + a^2 \cos^2 x)^{1/2} dx = \int 1 dx = x. \qquad (28)$$

In the same way, Eq.(16) is expressed as

$$\rho = \frac{1}{(a^2 - a^2 \cos^2 x)^{1/2}} = \frac{1}{a}. \qquad (29)$$

Then,

$$\log \rho = \underline{C}. \qquad (30)$$

This result shows that the curve is a circular arc having some radius of curvature, and the "gradient" of the *C curve* becomes infinity, i.e., vertical.

[Equiangular spiral]

This curve is defined by

$$\left. \begin{array}{l} x = ae^{b\theta} \cos \theta \\ y = ae^{b\theta} \sin \theta \end{array} \right\}. \qquad (31)$$

This curve is second times continuously differentiable, and as this curve is expressed by the parameters $(x = \phi(t), y = \varphi(t))$, it is possible to calculate the radius of curvature ρ by

$$\rho = \frac{(\phi'(t)^2 + \varphi'(t)^2)^{3/2}}{|\phi'(t)\varphi''(t) - \phi''(t)\varphi'(t)|}. \qquad (32)$$

Substituting Eq.(31) in Eq.(32), we obtain

$$\rho = \frac{(a^2 e^{2b\theta}(1 + b^2))^{3/2}}{a^2 e^{2b\theta}(1 + b^2)} = ae^{b\theta}(1 + b^2)^{1/2}. \qquad (33)$$

The length of segmental curve s in an 'interval of radius of curvature' is defined as

$$s = \int (\phi'(t)^2 + \varphi'(t)^2)^{1/2} dx. \qquad (34)$$

And, the relation between s and θ is expressed by

$$s = \int ds = \int ae^{b\theta}(1 + b^2)^{1/2} d\theta = \frac{ae^{b\theta}(1 + b^2)^{1/2}}{b}. \qquad (35)$$

Substituting Eq.(33) in Eq.(35), we obtain

$$s = \frac{\rho}{b}. \qquad (36)$$

When both sides are expressed logarithmically,

$$\log \rho = \log s + \log b, \qquad (37)$$

that is,

$$Y = \underline{1} \times X + C. \qquad (38)$$

[Logarithmic curve]

This curve is defined by

$$y = a \log x. \qquad (39)$$

Since this curve is second times continuously differentiable, it is possible to calculate the radius of curvature ρ by Eq.(2). Substituting Eq.(39) in Eq.(2), we obtain

$$\rho = \frac{(1 + a^2/x^2)^{3/2}}{|-a/x^2|}, \qquad (40)$$

that is,

$$a\rho = x^2(1 + a^2/x^2)^{3/2}. \qquad (41)$$

The length of segmental curve s in an 'interval of radius of curvature' is defined as

$$s = \int (1 + a^2/x^2)^{1/2} dx. \tag{42}$$

As a logarithmic curve in a region where $x \gg a > 1$ is adopted in design generally, $1 + a^2/x^2 \cong 1$, and Eq.(42) is expressed as

$$s = \int ds = \int dx = x. \tag{43}$$

Substituting Eq.(43) in Eq.(41), we obtain

$$a\rho = x^2(1 + a^2/x^2)^{3/2} = x^2 = s^2, \tag{44}$$

because $1 + a^2/x^2 \cong 1$. When both sides are expressed logarithmically,

$$\log a + \log \rho = 2 \log s, \tag{45}$$

that is,

$$Y = \underline{1/2} \times X + C. \tag{46}$$

As mentioned above, the "gradient" of several *C curves* could be calculated mathematically. When a *C curve* has some constant "gradient", it is possible to estimate the mathematical expression of the sample curve.

4. Analysises of sample curves

We analyzed more than one hundred sample curves drawn by expert car designers by using this quantitative analysis method, and studied what characteristics of a curve designer controlled to make an aesthetic curve.

As a result, we could classify these curves into four typical types. Here, we took a car as the motif in this study, because we confirmed that the appearance of a car is important for potential buyers, and hence, car designers were very sensitive to the curve.

Four typical types of "logarithmic distribution diagram of curvature" for curves are shown in Figures 7-10. Let us discuss these curves as follows.

[Analysis of curve 1]

This curve is the section line of a bonnet-hood of a Japanese car drawn by a designer. The result of analysis by our method is shown in Figure 7. From this result, we estimated he attempted to draw a curve as *C curve* consisting of a straight-line of "gradient"=1/2. It can be considered that this curve is roughly a logarithmic curve. However, we also confirmed that this curve contains ' noise' caused by hand drawing. Most curves of Japanese cars belong to this type.

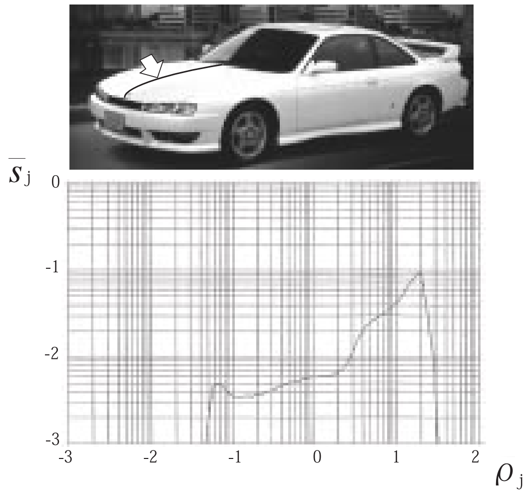

Figure 7. A result of the analysis of curve 1.

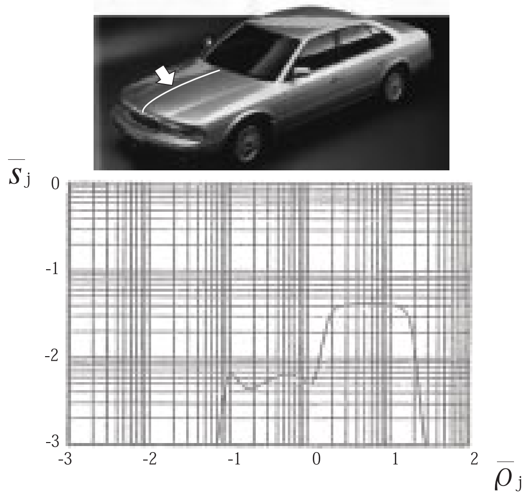

Figure 8. A result of the analysis of curve 2.

[Analysis of curve 2]

This curve is the section line of a bonnet-hood of a Japanese car drawn by a designer. The result of analysis by our method is shown in Figure 8. From this result, we estimated he attempted to draw the curve as *C curve* consisting of two straight-lines of "gradient"=0. However, we also confirmed that this curve contains ' noise' caused by hand drawing.

[Analysis of curve 3]

This curve is the section line of a bonnet-hood of an Italian car drawn by a designer. The result of analysis by our method is shown in Figure 9. From this result, we estimated he attempt to draw the curve as *C curve* consisting of a straight-line of "gradient"= -1. It can

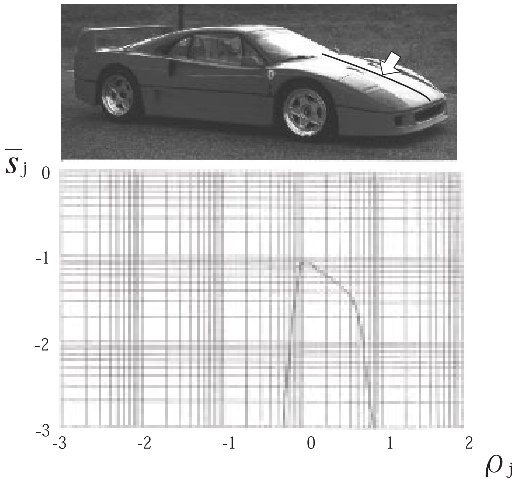

Figure 9. A result of the analysis of curve 3.

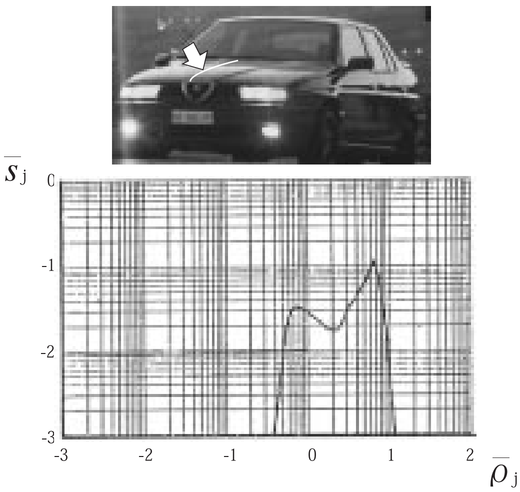

Figure 10. A result of the analysis of curve 4.

be considered that this curve is roughly a part of a sine curve.

[Analysis of curve 4]

This curve is the outline of a bonnet-hood of an Italian car drawn by a designer. The result of analysis by our method is shown in Figure 10. From this result, we gathered the designer attempted to draw the curve as *C curve* consisting of two straight-lines of "gradient"= -1 and "gradient"= 2/3. It can be considered that this curve is roughly combined with a parabola and a part of a sine curve. However, we also confirm that that curve contains ' noise' caused by hand drawing.

Through these analyses of four typical types of curves, we can abstract three common points.

1) A designer attempts to draw curves having the property that the *C curves* consist of one or two straight-lines of some "gradient" in the "logarithmic distribution diagram of curvature". However, as mentioned above, the *C curves* contained ' noise' and were not smooth. In other words, we confirm it is very difficult for a designer to draw a curve having the property that *C curve* consists of one or two straight-lines, i.e., having a precise self-affine property.

2) Most *C curves* have one or two straight-lines with a ' plus' or 'minus' or '0' "gradient" sign. This fact means that the designer aims to draw the curve as making the simplest shape and having a self-affine property. In other words, we can conclude that the designer recognizes a curve having a self-affine property as an aesthetic curve.

3) Curves 1 and 2 are Japanese car's, and curves 3 and 4 are Italian car's. We compared the two, and found that they had each characteristic inherent in nationality. An Japanese car designer prefer the 'decelerating' curve like parabola. An Italian car designer prefer the 'accelerating' curve as *C curve* consisting of a straight-line of "gradient"= -1.

5. Systematization of curves and applications

In this section, we systematize curves for design on the basis of our results above, and discuss impressions of these curves.

We confirmed that the impression of a curve was different according to what "gradient" sign is (i.e., 'plus' or 'minus' or '0') mainly by analyzing many samples. In fact the impressions of two curves are the same irrespective of "gradient", if "gradient" signs are both 'plus'.

Consequently, we could systematize and classify curves into the five types as shown in Figure 11. Figure 11 also shows the impressions of curves. Four of the five types are generalized by using the results above. The type of 'plus + minus' was not abstracted from the curves of actual cars. However we include the type of 'plus + minus', because the sine curve belongs to this type, and we confirm a curve of this type must be useful. Here, we shall call a curve as its *C curve* with one straight-line a 'monotonic-rhythm curve' and that with two straight-lines a 'compound-rhythm curve'. Furthermore, we developed a method to get any curve among the five types(any set of points representing the locus of a curve) having any volume, mathematically [6, 7]. The curves shown in Figure 11 were made by this method. In addition, we couldn't abstract a *C curve* with more than three straight-lines of some constant "gradient". On the basis of this fact, we consider that

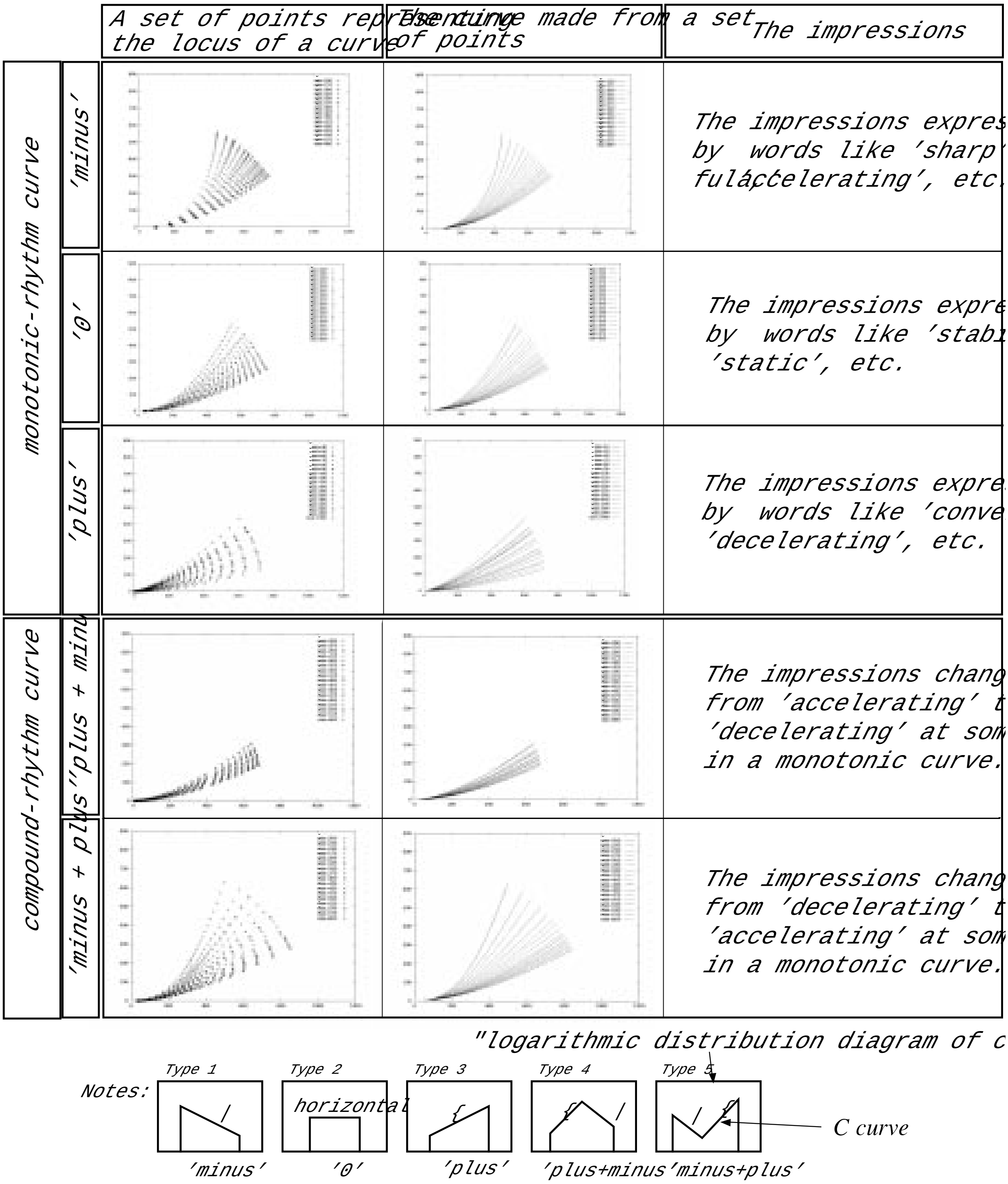

Figure 11. A systematization of curves and their impressions: The 5 basic types of an aesthetic curve.

Figure 12. 'drawing-curves' of 'minus' type.

Figure 13. 'drawing-curves' of '0' type.

these curves are not aesthetic. Actually, the curvature change is complex.

Accordingly, we consider the aesthetic curve(limited to a monotonic curve) a designer aims to draw as belongings to one of the five types. Furthermore, we propose a new method, where the five types of curves shown in Figure 11 are used as 'visual language' [9], and an aesthetic curve is made by using that 'visual language'.

We made 'drawing-curves' from above curves as 'visual language'(see Figure 12-14). These ' drawing-curves' can be used as a 'common language' between the designer, the modeler, and the operator of CAD systems for communicating the ' design intent'. We applied 'drawing-curves' to car design. Through a lot of simulations, we confirmed that 'drawing-curves' was a very useful tool for design work, i.e., we can make a curve and a shape efficiently by using that 'visual language'.

Figure 14. 'drawing-curves' of 'plus' type.

6. Conclusions

The fruits of this study are as follows. 1)When the designer defines a curve, the curvature change and the volume are controlled as the major characteristics of the curve. We proposed a mathematical method for quantifying the characteristics of the curve. By using the method, we can analyze the curve from a new aspect not discussed in former studies. 2)A lot of curves adopted into actual cars, were analyzed by this quantitative analysis method. As a result, we confirmed the designer aimed to draw the curve as having a self-affine property, i.e., the designer thought that such a curve was aesthetic. 3)We proposed five types of curves with self-affine property as 'visual language'. We made 'drawing-curves' from these curves as 'visual language'. We applied 'drawing-curves' to car design. As a result, we confirmed that 'drawing-curves' was a very useful tool for design work.

We have a few subjects for future work as follows. At present, we are implementing our algorithm on a computer and developing a prototype system for making curves by using our 'visual language' in CAD systems. For this system, it is important to develop a good GUI(graphical user interface) of operations for making a curve by using our 'visual language'. For that purpose, we have to analyze works design curves and surfaces. Further, we have to verify possibility of designer's accepting.

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